Consider the static, spherically symmetric metric

$$ds^2 = -e^{\nu(r)}dt^2 + e^{a(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta \; d\phi^2). \tag{1}$$

The non vanishing Christoffel symbols are

$$\Gamma^t_{tr} = \frac{1}{2}\nu'(r) \tag{2}$$

$$\Gamma^r_{tt} = \frac{1}{2}\nu'(r)e^{\nu(r)-a(r)} \qquad \Gamma^r_{rr} = \frac{1}{2}a'(r) \tag{3}$$

$$\Gamma^r_{\theta\theta} = -re^{-a(r)} \qquad \Gamma^r_{\phi\phi} = -r\sin^2 e^{-a(r)} \tag{4}$$

$$\Gamma^\theta_{\theta r} = \frac{1}{r} \qquad \Gamma^\theta_{\phi\phi} = -\cos\theta\sin\theta \tag{5}$$

$$\Gamma^\phi_{\phi r} = \frac{1}{r} \qquad \Gamma^\phi_{\phi\theta} = \cot\theta. \tag{6}$$

The trace terms are

$$\Gamma^\sigma_{t\sigma} = 0 \qquad \Gamma^\sigma_{r\sigma} = \frac{2}{r} + \frac{1}{2}(\nu'(r) + a'(r)) \tag{7}$$

$$\Gamma^\sigma_{\theta\sigma} = \cot\theta \qquad \Gamma^\sigma_{\phi\sigma} = 0. \tag{8}$$

The Ricci tensor is defined by

$$R_{\mu\nu} = \Gamma^\rho_{\mu\nu,\rho} - \Gamma^\rho_{\mu\rho,\nu} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho} - \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\sigma\nu}. \tag{9}$$

Its components are

$$\begin{aligned}
R_{tt} &= e^{\nu(r)-a(r)}\left(\frac{1}{2}\nu''(r) + \frac{1}{4}\nu'(r)^2 + \frac{1}{r}\nu'(r) - \frac{1}{4}a'(r)\nu'(r)\right) \\
R_{rr} &= -\frac{1}{2}\nu''(r) - \frac{1}{4}\nu'(r)^2 + \frac{1}{4}a'(r)\nu'(r) + \frac{1}{r}a'(r) \\
R_{\theta\theta} &= 1 - e^{-a(r)} + \frac{1}{2}ra'(r)e^{-a(r)} - \frac{1}{2}r\nu'(r)e^{-a(r)} \\
R_{\phi\phi} &= \sin^2\theta R_{\theta\theta} \\
R_{\mu\nu} &= 0 \;\text{ if } \mu \neq \nu.
\end{aligned}$$

The Ricci scalar is

$$\begin{aligned}
R = g^{\mu\nu}R_{\mu\nu} &= -\nu''(r)e^{-a(r)} - \frac{1}{2}\nu'(r)^2e^{-a(r)} + \frac{1}{2}a'(r)\nu'(r)e^{-a(r)} \\
&\quad + \frac{2}{r^2} - \frac{2e^{-a(r)}}{r^2} + \frac{2}{r}a'(r)e^{-a(r)} - \frac{2}{r}\nu'(r)e^{-a(r)}.
\end{aligned}$$

The Einstein tensor is defined by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \tag{10}$$

Its four components are

$$G_{tt} = \frac{1}{r^2} e^{\nu(r)} \frac{d}{dr} \left(r - r e^{-a(r)} \right) \tag{11}$$

$$G_{rr} = \frac{1}{r^2} \left(1 + r\nu'(r) - e^{a(r)} \right) \tag{12}$$

$$G_{\theta\theta} = r^2 e^{-a(r)} \frac{1}{2} (\nu''(r) - \frac{1}{r} a'(r) + \frac{1}{r} \nu'(r) + \frac{1}{2} \nu'(r)^2 - \frac{1}{2} a'(r) \nu'(r)) \tag{13}$$

$$G_{\phi\phi} = \sin^2 \theta G_{\theta\theta}. \tag{14}$$

The Einstein tensor has vanishing covariant derivative and therefore implies energy-momentum conservation,

$$\nabla_\nu T^{\mu\nu} = 0. \tag{15}$$

For a perfect fluid in a static, spherically symmetric spacetime the energy-momentum tensor has the form

$$T_{\mu\nu} = (\rho(r) + P(r)) U_\mu U_\nu + P(r) g_{\mu\nu} \tag{16}$$

where 4-velocity $U_t = -e^{\nu(r)/2}$ is given for a fluid at rest. Conservation of this quantity gives four equations of motion. Because of symmetries, only the radial component $\mu = r$ does not equal zero. By using the Christoffel symbols derived above, it is found that

$$0 = (\rho(r) + P(r)) \frac{\nu'(r)}{2} e^{-a(r)} + P'(r) e^{-a(r)}$$

which gives

$$2P'(r) = -(P(r) + \rho(r)) \nu'(r). \tag{17}$$